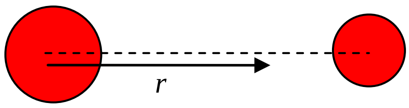
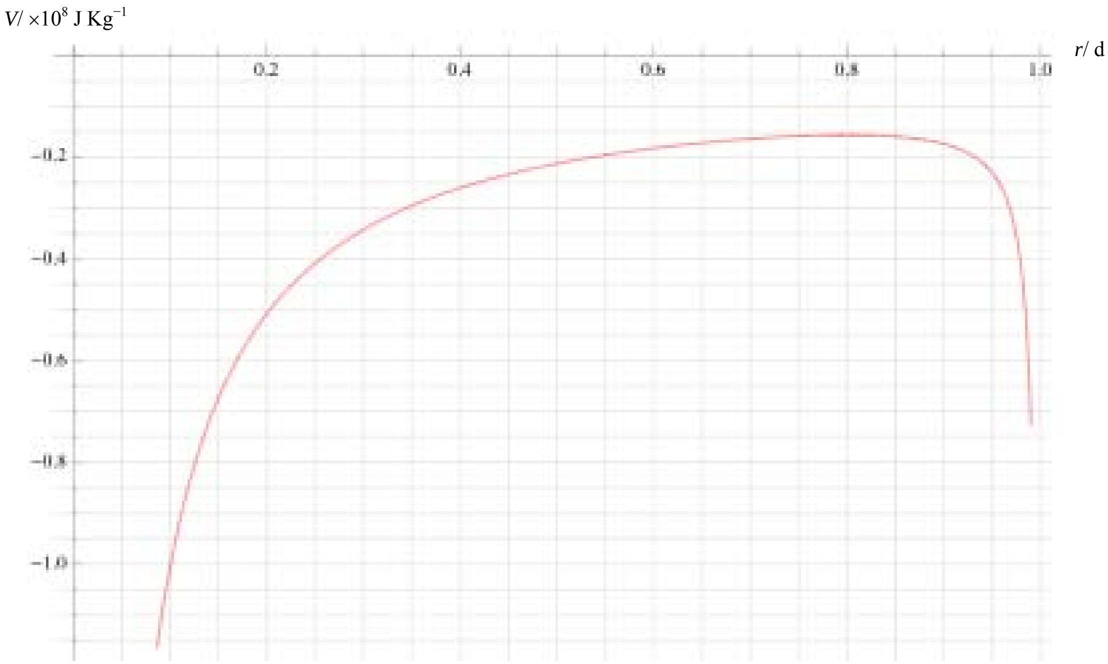
Extension Worksheet – Topic 2, Worksheet 6

1 The diagram shows a planet and a smaller moon in orbit around the planet.



The distance between the planet and the moon is $d = 4.0 \times 10^7$ m. The graph shows the variation of the gravitational potential created by two bodies with $\frac{r}{d}$ where r is the distance from the centre of the planet (on the left).



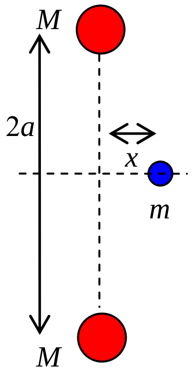
a State and explain why at a distance of $r = 3.2 \times 10^7$ m from the centre of the planet the gravitational field strength is zero. [3]

b Use the answer in **a** to deduce the ratio of the mass of the planet to the mass of the moon. [2]

c A probe is released from the point with $r = 3.2 \times 10^7$ m so that it begins to move towards the planet (with negligible initial speed). Calculate the speed of the probe when it reaches the point a distance at $r = 0.8 \times 10^7$ m. [2]

d Draw a sketch graph (no numbers required) to show how the gravitational field strength varies with $\frac{r}{d}$ for points on the line in between the two masses. [2]

2 The centres of two fixed, identical spherical bodies of mass M are separated by a distance $2a$. A particle of mass m is placed a distance x from the midpoint of the bodies along the perpendicular bisector of the line joining the masses as shown.



a Calculate the magnitude and direction of the gravitational field strength created by the two bodies at the position of the particle. [3]

b Show that when x is very small compared to a, the particle will execute simple harmonic oscillations. [2]

c Determine the period of oscillations. [2]

3 A probe is launched from the surface of a planet where the gravitational potential is $V_0 = -2.8 \times 10^9$ J kg^{-1}. The radius of the planet is $R = 4.5 \times 10^5$ m.

a Show that the escape speed from the surface of the planet is given by $v_{\text{esc}} = \sqrt{-2V_0}$. [2]

b Calculate the escape velocity from this planet. [1]

c A probe is launched from the surface of the planet with a speed that is half the escape speed. Show that the height from the surface of the planet the probe will get to, before falling back to the planet, is $h = 1.5 \times 10^5$ m. [3]

4 A probe is launched from the surface of a planet of mass M and radius R. The kinetic energy of the probe at launch is $E_K = \frac{3GMm}{4R}$. The probe eventually settles into a circular orbit around the planet.

a Show that the total energy of a probe in orbit at an orbit radius r is $E = -\frac{GMm}{2r}$. [2]

b Calculate the radius of the probe's orbit in terms of R. [2]

c The probe experiences a small frictional force from the atmosphere of the planet. State and explain the effect of this force on (i) the orbit radius and (ii) the speed of the probe. [3]

